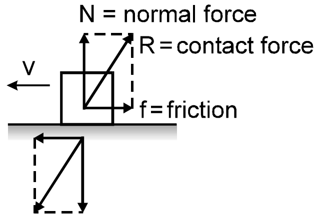
FRICTION

1. FRICTION :

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force of the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (figure. The perpendicular component is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

Therefore if R is contact force then

$$R = \sqrt{f^2 + N^2}$$

2. REASONS FOR FRICTION

1. Inter-locking of extended parts of one object into the extended parts of the other object.

2. Bonding between the molecules of the two surfaces or objects in contact.

3. Friction force is of two types.

a. Kinetic b. Static

3. (a) KINETIC FRICTION FORCE

Kinetic friction exists between two contact surfaces only when there is **relative motion** between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

Direction of kinetic friction on an object

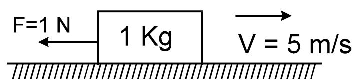
It is opposite to the velocity of the object with respect to the other object in contact considered.

Note that its direction is not opposite to the force applied it is opposite to the motion of the body considered which is in contact with the other surface.

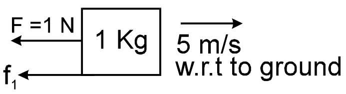
Ex. Find the direction of kinetic friction force

(a) on the block, exerted by the ground.

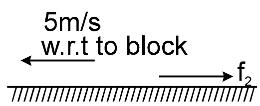
(b) on the ground, exerted by the block.



Sol. (a)



(b)



where f_1 and f_2 are the friction forces on the block and ground respectively.

Ex. The correct relation between magnitude of f_1 and f_2 is

(A) $f_1 > f_2$ (B) $f_2 > f_1$ (C) $f_1 = f_2$ (D) not possible to decide due to insufficient data.

Sol. By Newton's third law the above friction forces are action-reaction pair and equal but opposite to each other in direction. Hence (C).

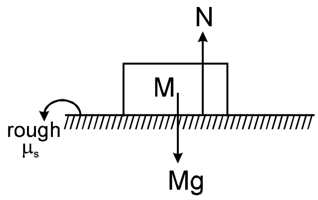
Also note that the direction of kinetic friction has nothing to do with applied force F.

3. (b) STATIC FRICTION

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

For example consider a bed inside a room ; when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the opposite direction of the applied force.

Ex. What is value of static friction force on the block?



Sol. In horizontal direction as acceleration is zero. Therefore $\Sigma F = 0$. $\therefore f = 0$

Direction of static friction force :

The static friction force on an object is opposite to its impending motion relative to the surface.

Following steps should be followed in determining the direction of static friction force on an object.

(i) Draw the free body diagram with respect to the other object on which it is kept.

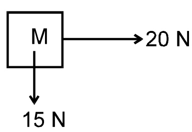
(ii) Include pseudo force also if contact surface is accelerating.

(iii) Decide the resultant force and the component parallel to the surface of this resultant force.

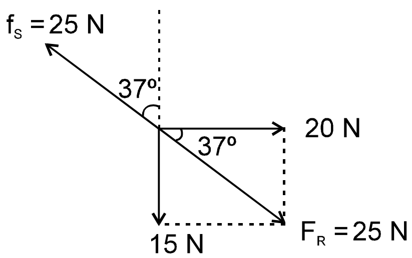
(iv) The direction of static friction is opposite to the above component of resultant force.

Note : Here once again the static friction is involved when there is no relative motion between two surfaces.

Ex. In the following figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.



Sol. In the above problem we first draw the free body diagram to find the resultant force.



As the object doe not move this is not a case of limiting friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure by f_s. Its magnitude is equal to 25 N.

4. MAGNITUDE OF KINETIC AND STATIC FRICTION

Kinetic friction :

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.** If the surfaces are smooth μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

Static friction :

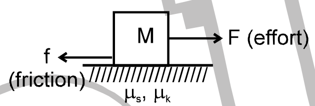
The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. **This means it is a variable and self adjusting force. However it has a maximum value called limiting friction.**

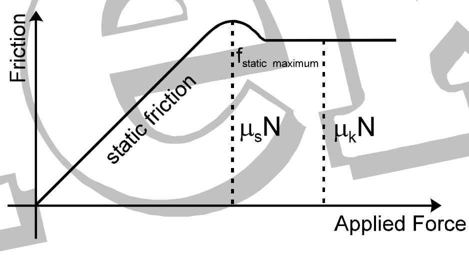
$$f_{max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0 \le f_s \le f_{smax}$$

Here μ_s and μ_k are proportionality constants. μ_s is called coefficient of static friction and μ_k is called coefficient of kinetic friction. They are dimensionless quantities independent of shape and area of contact . It is a property of the two contact surfaces. $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned then $\mu_s = \mu_k$ can be taken. Value of μ can be from 0 to ∞.





Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

Ex.1 The coefficient of static friction between a block of mass m and an incline of angle θ is 0.3. (a) What can be the maximum angle θ of the incline with the horizontal so that the block does not slip on the plane ? (b) If the incline makes an angle $\theta/2$ with the horizontal, find the friction force on the block.

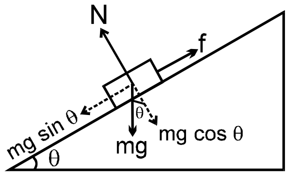
Sol. The situation is shown in free body diagram.

(a) The forces acting on the block are

(i) mg, exerted downward by the earth,

(ii) N, normal contact force by the incline, and

(iii) f, friction force f parallel to the incline up the plane, by the incline.



As the block is at rest, these forces should add up to zero. Also, since θ is the maximum angle to prevent slipping, this is a case of limiting equilibrium therefore $f = \mu_s N$

Taking components perpendicular to the incline,

$N - mg\cos\theta = 0$

or $N = mg\cos\theta$ .... (i)

Taking components parallel to the incline,

$f - mg\sin\theta = 0$

or $f = mg\sin\theta$

or $\mu_s N = mg\sin\theta$. .... (ii) ($\because$ f can have a maximum value of μ_s N when θ is increased)

Dividing (ii) by (i) $\mu_s = \tan\theta$

or $\theta = \tan^{-1}\mu_s$ $\theta = \tan^{-1}(0.3)$.

(b) If the angle of incline is θ/2, **the equilibrium is not limiting**, and hence the **force of static friction f is less than μ_sN.** To know the value of f, we proceed as in part (a) and get the equations.

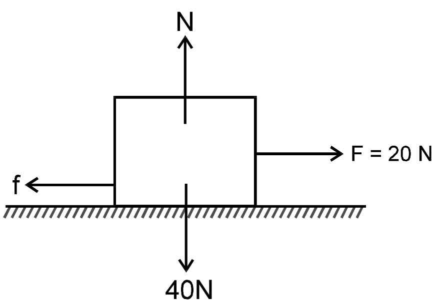
$N = mg\cos(\theta/2)$

and $f = mg\sin(\theta/2)$.

Thus, the force of friction is mg sin (θ/2).

Ex.2 A horizontal force of 20N is applied to a block of mass 4kg resting on a rough horizontal table. If the block does not move on the table, how much frictional force the table is applying on the block ? What can be said about the coefficient of static friction between the block and the table ? Take $g = 10\ m/s^2$.

Sol. The situation is shown in free body diagram.



The forces on the block are :

(a) 40 N, downward by the Earth,

(b) N, normal force upward by the table,

(c) F = 20 N, applied force,

(d) f, friction force towards left by the table.

As the block is at rest, these forces should add up to zero. Balancing the forces in horizontal and vertical directions as $a_x = 0$ and $a_y = 0$.

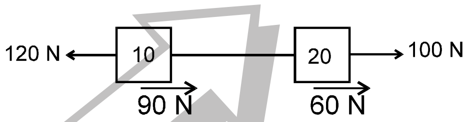
$$f = 20 \text{ N and } N = 40 \text{ N.}$$

Thus, the table exerts a friction (static) force of 20 N on the block in the direction opposite to the applied force. Since there is no relative motion exists hence friction is static.

$$f \le \mu_s N, \quad \text{or,} \quad \mu_s \ge f / N \quad \text{or,} \quad \mu_s \ge 0.5$$

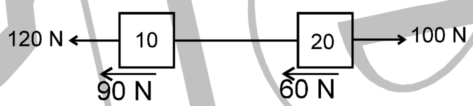
Ex. Find the tension in the string in situation as shwon in the figure below. Forces 120 N and 100 N start acting when the system is at rest.

Sol. (i) Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore the assumption is not valid.



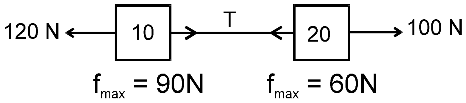
Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

(ii) Similarly let us assume that system moves towards right.

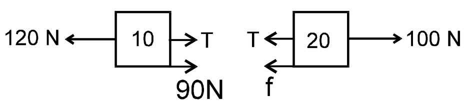


Above assumption is also not possible as net force on the system is towards left in this situation. Hence assumption is again not valid.

Therefore it can be concluded that the system is stationary.



Assuming that the 10 kg block reaches limiting friction first then using FBD's.

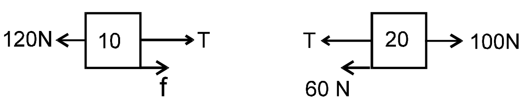


$120 = T + 90 \qquad \Rightarrow \qquad T = 30$ N

Also $T + f = 100$

$\therefore\ 30 + f = 100 \qquad \Rightarrow \qquad f = 70$ N which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have



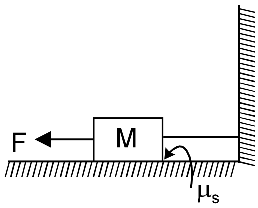
$T + 60 = 100\ N \Rightarrow \quad T = 40\ N$

Also $\quad f + T = 120\ N \quad \Rightarrow \quad f = 80\ N$

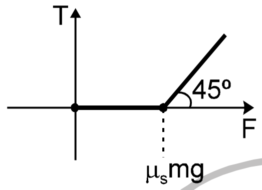
This is acceptable as static friction at this surface should be less than 90 N.

Hence the tension in the string is $\quad T = 40\ N$.

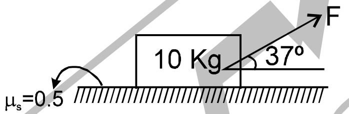
Ex. In the following figure force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s.



Sol. As the external force F is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s mg$, the further increase in F is transferred to the string.

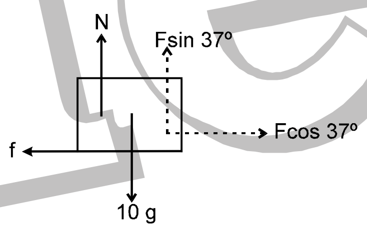


Ex.3



Force F is gradually increased from zero. Determine whether the block will first slide or lift up?

Sol.



There are minimum magnitude of forces required both in horizontal and vertical direction either to slide on lift up the block. The block will first slide on lift up will depend upon which minimum magnitude of force is lesser.

For vertical direction to start lifting up

$F \sin 37º + N - Mg \geq 0.$

N becomes zero just lifting condition.

$F_{lift} \geq \dfrac{10g}{3/5} \qquad \therefore \quad F_{lift} \geq \dfrac{500}{3} N$

For horizontal direction to start sliding $\qquad F \cos 37 \geq \mu_s N$

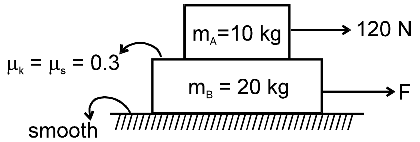
$F \cos 37º > 0.5\ [\ 10g - F \sin 37º]$ $\qquad (\because\ N = 10\ g - F \sin 37º)$

Hence $F_{slide} > \dfrac{50}{\cos 37º + 0.5 \sin 37º}$

$F_{slide} > \dfrac{500}{11} N \quad \Rightarrow \quad F_{lift} > \dfrac{500}{3} N. \quad \Rightarrow\ F_{slide} < F_{lift}$

Therefore the block will begin to slide before lifting.

Ex. In the figure given below force F applied horizontally on lower block, is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Take g = 10 m/s²).



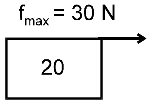
Ans. In the above situation we see that the maximum possible value of friction between the blocks is $\mu_s m_A g = 0.3 \times 10 \times 10 = 30$ N.

Case (i) When F = O.

Considering that there is no slipping between the blocks the acceleration of system will be

$$a = \frac{120}{20+10} = 4 \text{ m/s}^2$$

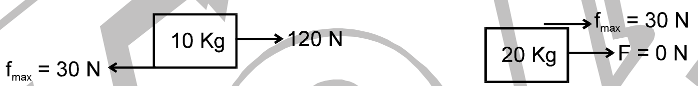
But the maximum acceleration of B can be obtained by the following force diagram.



$a_B = \frac{30}{20} = 1.5$ m/s² ($\because$ only friction force by block A is responsible for producing acceleration in block B)

Because 4 > 1.5 m/s² we can conclude that the blocks do not move together.

Now drawing the F.B.D. of each block, for finding out individual accelerations.

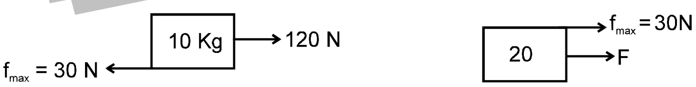


$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2 \text{ towards right}$$

$$a_B = \frac{30}{20} = 1.5 \text{ m/s}^2 \text{ towards right.}$$

Case (ii) F is increased from zero till the two blocks just start moving together.

As the two blocks move together the friction is static in nature and its value is limiting. FBD in this case will be



$$a_A = \frac{120-30}{10} = 9 \text{ m/s}^2$$

$$a_B = \frac{F+30}{20} = a_A \quad \Rightarrow \quad \frac{F+30}{20} = 9 \quad \therefore \quad F = 150 \text{ N}$$

Hence when 0 < F < 150 N the blocks do not move together and the friction is kinetic. As F increases acceleration of block B increases from 1.5 m/s².

At F = 150 N limiting static friction start acting and the two blocks start moving together.

Case (iii) When F is increased above 150 N.

In this scenario the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case the FBD is as follows

$$a_A = a_B = \frac{120-f}{10} = \frac{F+f}{20}$$

$\therefore$ when friction force f gets reduced to zero the above accelerations become

$$a_A = \frac{120}{10} = 12 \text{ m/s}^2$$

$$a_B = \frac{F}{20} = a_A = 12 \text{ m/s}^2$$

$\therefore$ F = 240 N

Hence when $150 \le F \le 240$ N the static friction force continuously decreases from maximum to zero at F = 240 N. The accelerations of the blocks increase from 9 m/s^2 to 12 m/s^2 during the change of force F.

Case (iv) When F is increased again from 240 N the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. FBD at this juncture will be

The blocks move together therefore.

$$a_A = \frac{120+30}{10} = 15 \text{ m/s}^2$$

$$a_B = \frac{F-30}{20} = a_A = 15 \text{ m/s}^2 \qquad \therefore \frac{F-30}{20} = 15 \text{ m/s}^2$$

Hence F = 330 N.

Case (v) When F is increased beyond 330 N. In this case the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).

$\therefore$ a_A = 15 m/s^2 which is constant

$$a_B = \frac{F-30}{20} \text{ m/s}^2 \text{ where } F > 330 \text{ N.}$$

